Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Tuesday 13 November 2018 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1.   [Maximum mark: 6]

In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.
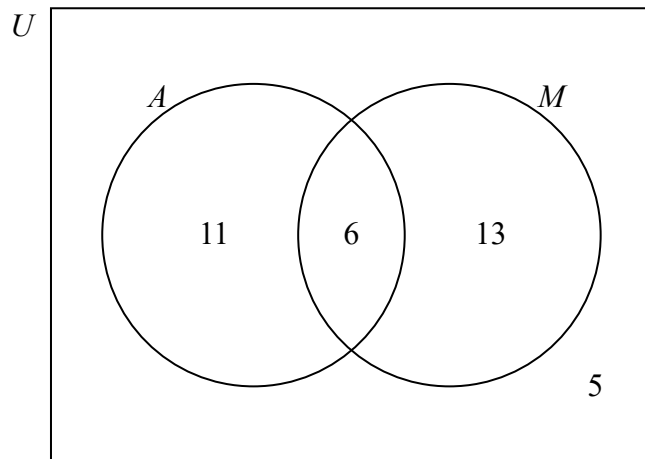
(a)   Write down the number of students in the group who take art class.   [2]

(b)   One student from the group is chosen at random. Find the probability that

(i)   the student does not take art class;

(ii)   the student takes either art class or music class, but not both.   [4]

(This question continues on the following page)

(Question 1 continued)

12EP03

2.   [Maximum mark: 6]

The following table shows the hand lengths and the heights of five athletes on a sports team.

Hand length (x cm)	21.0	21.9	21.0	20.3	20.8
Height (y cm)	178.3	185.0	177.1	169.0	174.6

The relationship between x and y can be modelled by the regression line with equation $y = ax + b$.

(a)   (i)   Find the value of a and of b.

(ii)   Write down the correlation coefficient. [4]

(b)   Another athlete on this sports team has a hand length of 21.5 cm. Use the regression equation to estimate the height of this athlete. [2]

3. [Maximum mark: 7]

Let $f(x) = \dfrac{6x-1}{2x+3}$, for $x \neq -\dfrac{3}{2}$.

(a) For the graph of f,

   (i) find the y-intercept;

   (ii) find the equation of the vertical asymptote;

   (iii) find the equation of the horizontal asymptote. [5]

(b) Hence or otherwise, write down $\displaystyle\lim_{x \to \infty}\left(\dfrac{6x-1}{2x+3}\right)$. [2]

4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v \, \text{m s}^{-1}$, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \leq t \leq 5$.

(a) Find when the particle is at rest. [2]

(b) Find the acceleration of the particle when $t = 2$. [2]

(c) Find the total distance travelled by the particle. [3]

5.   [Maximum mark: 6]

The sum of an infinite geometric sequence is 33.25. The second term of the sequence is 7.98. Find the possible values of r.

6. [Maximum mark: 7]

Consider the expansion of $\left(2x^4 + \dfrac{x^2}{k} \right)^{12}$, $k \neq 0$. The coefficient of the term in x^{40} is five times

the coefficient of the term in x^{38}. Find k.

7.   [Maximum mark: 6]

A communication tower, T, produces a signal that can reach cellular phones within a radius of $32\,\text{km}$. A straight road passes through the area covered by the tower's signal.

The following diagram shows a line representing the road and a circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on the road. Point S is $38\,\text{km}$ from the tower and $R\hat{S}T = 43°$.

diagram not to scale
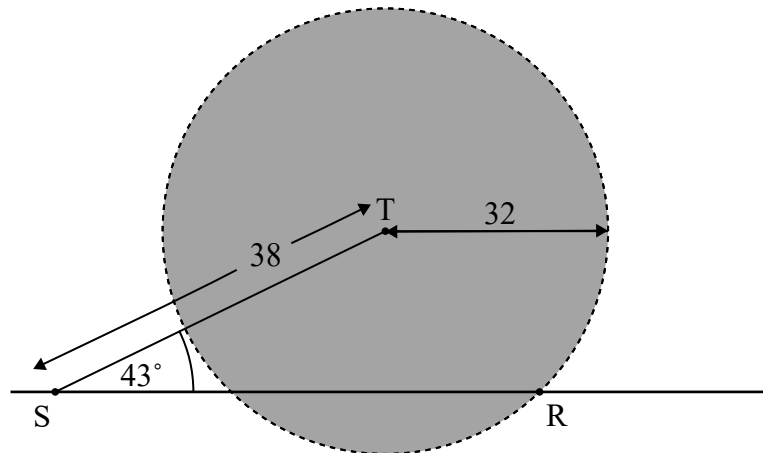
(a)   Let $SR = x$. Use the cosine rule to show that $x^2 - (76\cos 43°)\,x + 420 = 0$.   [2]

(b)   Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones.   [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8.   [Maximum mark: 16]

Consider the points $A(-3, 4, 2)$ and $B(8, -1, 5)$.

(a)   (i)   Find $\overrightarrow{AB}$.

       (ii)   Find $\left| \overrightarrow{AB} \right|$.   [4]

A line L has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point $C(5, y, 1)$ lies on line L.

(b)   (i)   Find the value of y.

       (ii)   Show that $\overrightarrow{AC} = \begin{pmatrix} 8 \\ -10 \\ -1 \end{pmatrix}$.   [5]

(c)   Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.   [5]

(d)   Find the area of triangle ABC.   [2]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A nationwide study on reaction time is conducted on participants in two age groups.
The participants in Group X are less than 40 years old. Their reaction times are
normally distributed with mean 0.489 seconds and standard deviation 0.07 seconds.

(a) A person is selected at random from Group X. Find the probability that their reaction
time is greater than 0.65 seconds. [2]

The participants in Group Y are 40 years or older. Their reaction times are normally
distributed with mean 0.592 seconds and standard deviation σ seconds.

(b) The probability that the reaction time of a person in Group Y is greater than 0.65 seconds
is 0.396. Find the value of σ. [4]

In the study, 38% of the participants are in Group X.

(c) A randomly selected participant has a reaction time greater than 0.65 seconds.
Find the probability that the participant is in Group X. [6]

(d) Ten of the participants with reaction times greater than 0.65 are selected at random.
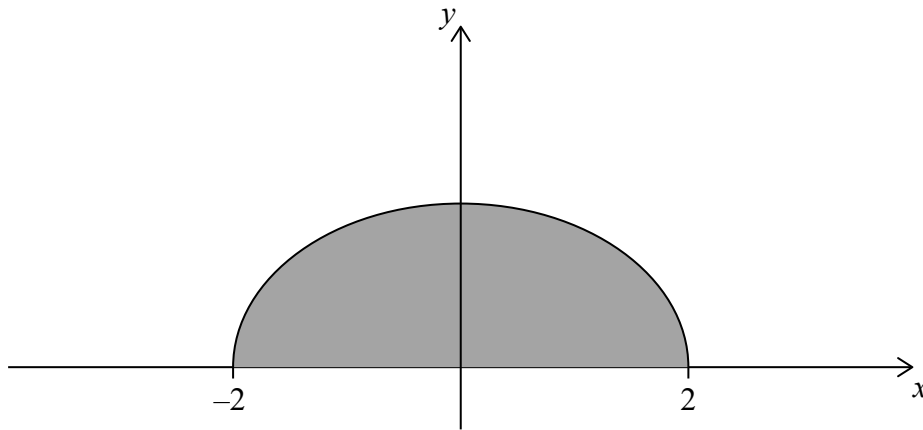Find the probability that at least two of them are in Group X. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

All lengths in this question are in metres.

Consider the function $f(x) = \sqrt{\dfrac{4 - x^2}{8}}$, for $-2 \le x \le 2$. In the following diagram, the shaded region is enclosed by the graph of f and the x-axis.

diagram not to scale

A container can be modelled by rotating this region by $360°$ about the x-axis.

(a)   Find the volume of the container. [3]

Water can flow in and out of the container.

The volume of water in the container is given by the function $g(t)$, for $0 \le t \le 4$, where t is measured in hours and $g(t)$ is measured in m^3. The rate of change of the volume of water in the container is given by $g'(t) = 0.9 - 2.5\cos(0.4t^2)$.

(b)   The volume of water in the container is increasing only when $p < t < q$.

   (i)   Find the value of p and of q.

   (ii)   During the interval $p < t < q$, the volume of water in the container increases by $k\,\text{m}^3$. Find the value of k. [6]

When $t = 0$, the volume of water in the container is $2.3\,\text{m}^3$. It is known that the container is never completely full of water during the 4 hour period.

(c)   Find the minimum volume of empty space in the container during the 4 hour period. [5]